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Handling of Chaos in Two Dimensional Discrete Maps

Anil Kumar Jain
Assistant Professor, Department of Mathematics
Barama College, Barama, Assam, Pincode-781346, India
(Email: jainanil965@gmail.com)

Abstract: Here we have considered a two dimensional discrete map for handling or controlling the chaos. This map follows period doubling route to chaos ahead of accumulation point. There are many methods available in the literature of dynamical systems for controlling of chaos for different models or maps. In this paper, we have applied the periodic proportional pulses technique which is proposed by N.P.Chau [2] to stabilise unstable periodic orbits embedded in the chaotic attractors of two dimensional non linear discrete maps.

Keywords:Fixed point, Period doubling bifurcation, accumulation point, chaos, Lyapunov exponents.

1. INTRODUCTION [1, 2, 3, 4, 6, 8, 9, 10]:

Handling or controlling of chaos is an essential part of the study of chaos theory. The idea of handling of chaos consists of stabilizing some of unstable periodic orbits, thus leading to regular and predictable behavior. The base of handling of chaos is that any chaotic attractor contains an infinite number of unstable periodic orbits. Then chaotic dynamics consists of a motion where the system state moves in the neighborhood of one of these orbits for a while, then falls close to a different unstable periodic orbit where it remains for a limited time, and so forth. Handling or controlling of chaos is the stabilization, by means of small system perturbations, of one of the unstable periodic orbits. The perturbation must be tiny, to avoid significant modification of the system's natural dynamics.

Chaos has been found in many models representing real world system. It is only last 20 years that scientists have come to realize the potential uses for systems displaying chaotic phenomena. They have attempted to remove chaos when applying the theory to physical models. Scientists are replacing the maxim "stability good, chaos bad" with "stability good, chaos better" for some systems. Lastly, they experimentally found that the existence of chaotic behaviour may even be desirable for certain systems [8].

In techniques of "controlling chaos", the breakthrough came with the work of Ott, Grebogi and Yorke in1990.They published a famous seminal paper and in that paper they introduced a new point of view in the development of techniques for the control of chaotic phenomena. Basic methods of controlling chaos along with several reprints of fundamental contributions may be found in the excellent text books of Kapitaniak [8].

The first experimental control was performed by Garfinkel et al. in 1992 on a biological system. They were able to stabilize arrhythmic behavior in eight out of eleven rabbit's heart using a feedback-control mechanism. The OGY algorithm was implemented theoretically by Lynch and Steele to control the chaos within a hysteresis cycle of a nonlinear bistable optical resonator using the real and imaginary parts of the electrical field amplitude. The proportional pulse method was introduced by Matias and Guemez. After that N.P. Chau discussed in a similar manner but gave some restrictions on the initial conditions by which chaos can be controlled. Here we have taken the method of periodic proportional pulse to control of chaos of two dimensional map [2, 10].

2. PERIODIC PROPORTIONAL PULSES METHOD AND TWO DIMENSIONAL MAPS:

2.1 Control Procedure:

Consider two dimensional discrete systems $x_{n+1} = k(x_n, y_n), y_{n+1} = l(x_n, y_n)$. The model can be written as $X_{n+1} = F(X_n)$ where X is a vector in R^2 . Let $G = KF^m$ where G is kicked map, K is a diagonal matrix having diagonal elements $k_1, k_2(say)$, F^m is the composite map of F up to m times. Here kicked is applied to the orbit of the composite map F^m once, every m steps, by multiply the x component of the dynamics by a factor k_1, the y component by a factor k_2 ,to control the dynamics. If $X = KF^m$i.e. X is a fixed point of G then the fixed point will be stable if the absolute value of the largest eigen value of the Jacobian matrix of G is less than 1 $(unity)$. In next step ,we find the value of k_1, k_2such that chaos is controlled.

2.2 Concern two dimensional model & control procedure:

Here we consider a two dimensional nonlinear discrete map. The two dimensional discrete map $T_{pb} = (k, l): R^2 \rightarrow R^2$ is defined by

$k(x, y) = -p + y + x^2,$

$l(x, y) = bx \dots\dots\dots.(A)$ where p, b are adjustable parameters.

The model develops chaos through period doubling route [5].Also the period-doubling cascade of the model accumulates at the accumulation point $1.1490 \dots\dots\dots\dots,$ after which chaos occur. In figure $2.2{:}\,1(a)$ the bifurcation graph and in figure $2.2{:}\,1(b)$ Lyapunov exponents graphs show the chaotic behaviour of the system. A system is chaotic if at least one of the Lyapunov exponents is positive. Clearly for the parameter $b = 0.2$ the system is chaotic.

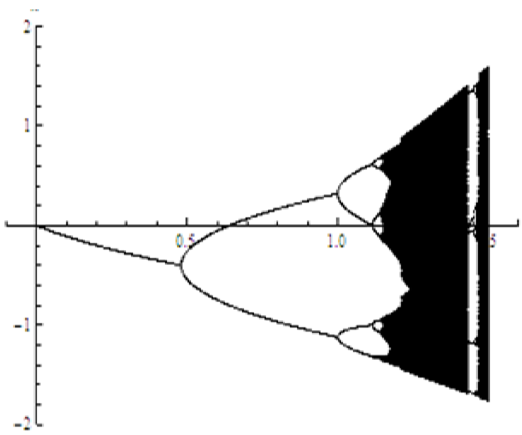

Figure $\mathbf{2.2{:}\,1(a)}$ Bifurcation graph of the model abscissa represents the value of the control parameter and ordinate represents f(x).

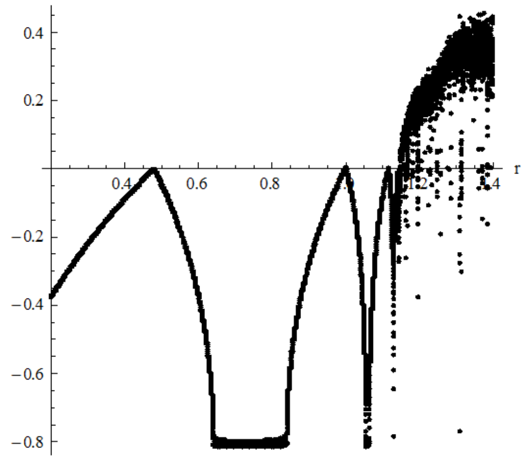

Figure $\mathbf{2.2{:}\,1(b)}$ Lyapunov exponents graph for our map taking control parameter $\boldsymbol{b = 0.2}$

Now for $m = 1$, the Jacobian of G will be

$$\begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \begin{bmatrix} 2x & 1 \\ b & 0 \end{bmatrix} = \begin{bmatrix} 2xk_1 & k_1 \\ bk_2 & 0 \end{bmatrix},$$

where $\begin{bmatrix} 2x & 1 \\ b & 0 \end{bmatrix}$ is the Jacobian of the function (A). The characteristic polynomial of the Jacobian matrix is

$$\gamma^2 - \gamma X + Y = 0$$

where

X =sum of diagonal elements= $2xk_1$,

Y = determinant= $-bk_1k_2$.

The eigen values of the Jacobian matrix for any point (x, y) are given by

$$\gamma = \frac{X \pm \sqrt{X^2 - 4Y}}{2} = xk_1 \pm \sqrt{x^2{k_1}^2 + bk_1k_2} \text{ -----(B)}$$

The fixed points will be stable if $|\gamma|$<1 If X=(x, y) is the fixed point of G, then

$k_1(-p + y + x^2) = x,$

$k_2bx = y$

and considering x≠0, y≠0 we have

$$k_1 = \frac{x}{-p+y+x^2}, \quad k_2 = \frac{y}{bx} \text{ -----(C).}$$

Now putting the values of k_1 , k_2 in (B) such that $|\gamma|$<1. After, with the help of suitable C-programming draw graphically the basin of attraction of the model for period one (i.e. $m = 1$) as follows. These basin of attraction points (x, y) of the model satisfies the $|\gamma|$<1 and the conditions of kicking factors k_1, k_2 .

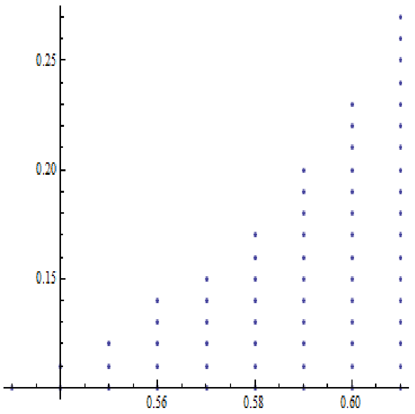

Figure 2.2:2(a) Basin of attraction of the model of period one

Now out of several number of points of the basin of attraction consider any particular point$(x = 0.600000, y = 0.190000)$ and find the values of k_1, k_2 from (C) such that $|\gamma|$<1.The values of $k_1, k_2, \gamma_1, \gamma_2$ are got (-1), (1.5833….), (-0.3916733…), (-0.8083266……) respectively. Again using control procedure with above k_1, k_2 we get graphical control representation as follows.

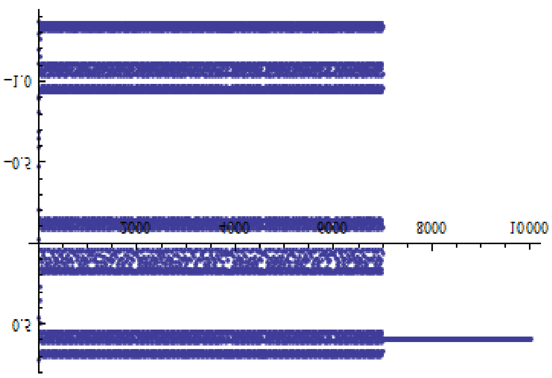

Figure2.2:2(b) Chaotic region up to 7000 iterations after which chaos is controlled by taking initial value of(x, y) from shaded portion of the fig 2.2:2(a)

In figure 2.2:2(b) up to 7000 iterations are done at the parameter p=1.15.

Now, for the other values of m, the Jacobian matrix is obtained as

$$\begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \begin{bmatrix} \frac{\partial x_m}{\partial x} & \frac{\partial x_m}{\partial y} \\ \frac{\partial y_m}{\partial x} & \frac{\partial y_m}{\partial y} \end{bmatrix},$$

where k_1, k_2 are kicking factors and can be expressed as

$$k_1 = \frac{x}{-p + y_{m-1} + x_{m-1}^2}, k_2 = \frac{y}{bx_{m-1}}$$

$, x_{m-1}$ is the first component of f^{m-1}, y_{m-1} is second component of f^{m-1}. By above procedure we get the basins of attraction and controlling chaos of the model for different periodic orbits using different values of m.

For $m = 2$, the basin of attr action is got as shown in figure 2.2:3(a). The shaded portion of the figure represents the point (x, y) .This point may be converted into stable fixed point by using suitable values of k_1, k_2

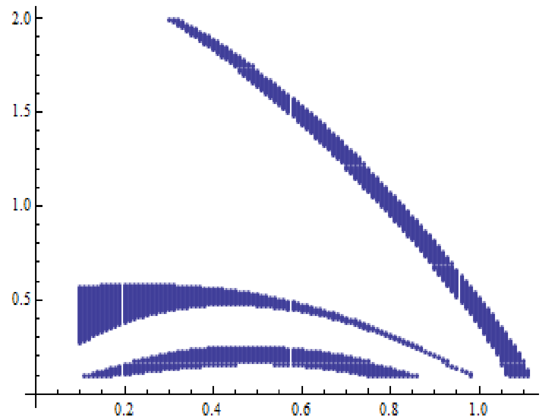

Figure 2.2:3(a) Basin of attraction of the model of period two

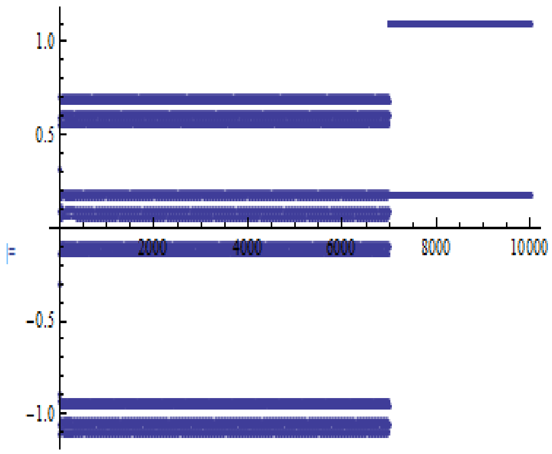

Figure2.2:3(b) Chaotic region up to 7000 iterations after which chaos is controlled by taking initial value of (x, y) from shaded portion of the figure 2.2:3(a)

For $\boldsymbol{m} = \mathbf{3}$,

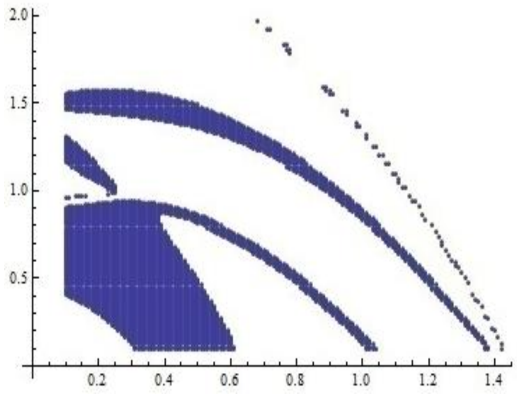

Figure2. 2: 4(a) Basin of attraction of the model of period three

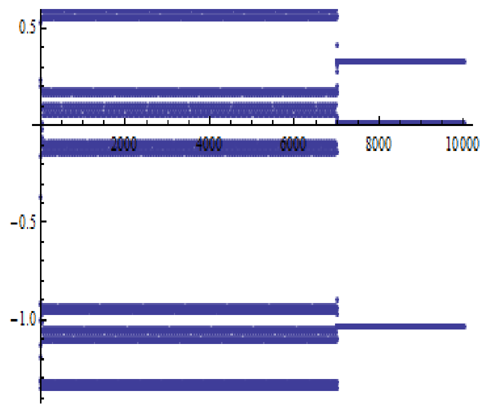

Figure 2. 2: 4(b) Chaotic region up to 7000 iterations after which chaos is controlled by taking initial value of (x, y) from shaded portion of the figure 2. 2: 4(a)

For $\boldsymbol{m} = \mathbf{4}$,

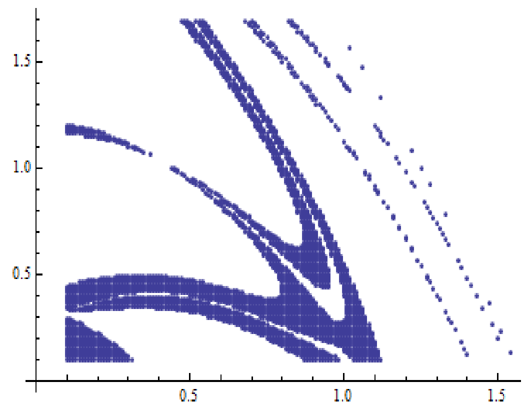

Figure 2. 2: 5(a) Basin of attraction of the model of period four

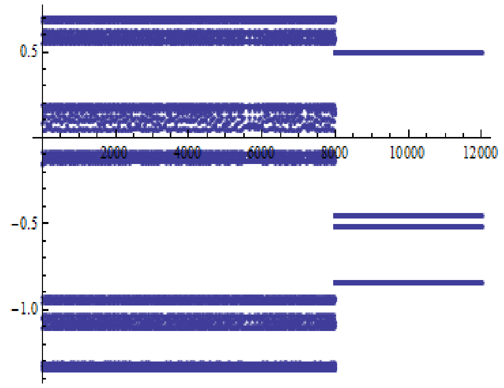

Figure 2.2:5 (b) Chaotic regions up to 7000 iterations after which chaos is controlled by taking initial value of (x, y) from shaded portion of the figure 2. 2: 5(a)

For $\boldsymbol{m} = \mathbf{8}$,

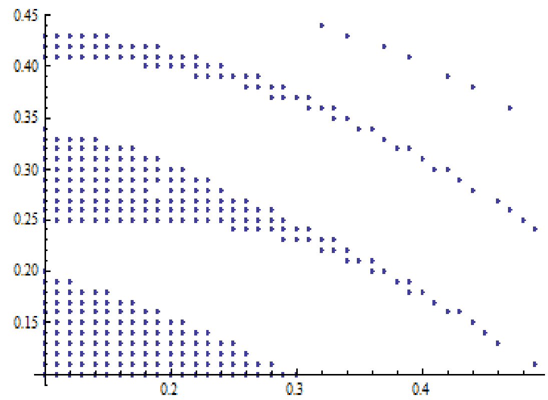

Figure2. 2: 6(a) Basin of attraction of the model of period eight

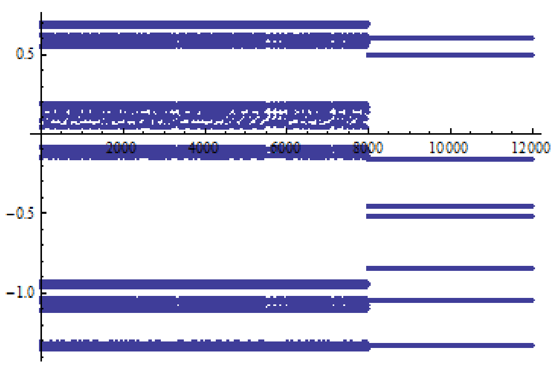

Figure 2. 2: 6(b) Chaotic regions up to 7000 iterations after which chaos is controlled by taking initial value of (x, y) from shaded portion of the figure 2. 2: 6(a)

For $m = 16$,

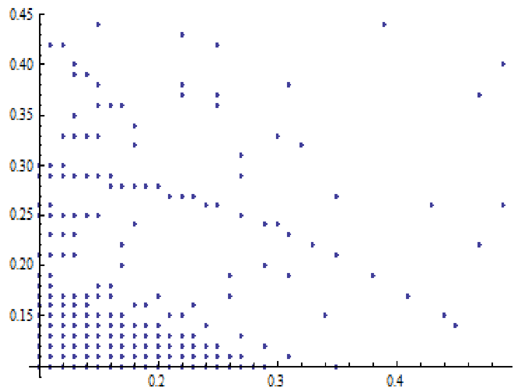

Figure2. 2: 7(a) Basin of attraction of the model of period sixteen

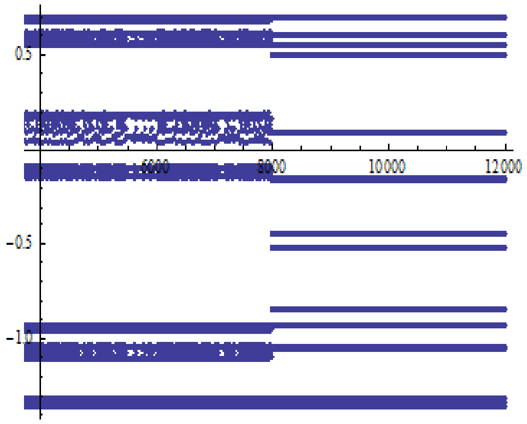

Figure : 2. 2: 7(b) Chaotic regions up to 7000 iterations after which chaos is controlled by taking initial value of (x, y) from shaded portion of the figure 2. 2: 7(a)

For $m = 32$,

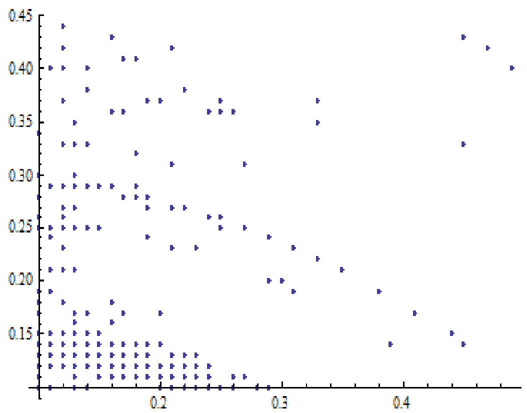

Figure2. 2: 8(a) Basin of attraction of the model of period thirty two

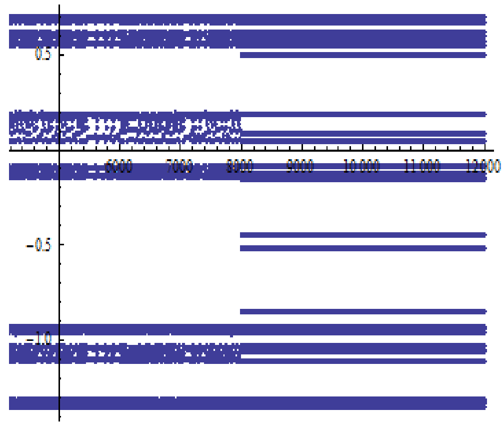

Figure 2. 2: 8(b) Chaotic regions up to 7000 iterations after which chaos is controlled by taking initial value of (x, y) from shaded portion of the figure 2. 2: 8(a)

3. CONCLUSION [7, 10]:

In conclusion, it can be said that an irregular orbit of any period can be controlled by the above technique. Also we can comment that chaos is very speculative. It is noticeable that chaotic behavior and its possible control help to get a new model about the behavior of complex system. In last few years, the implementation of the control algorithm has been carried out electronically using either digital signals or analog hardware. The hope for the future is that all optical processors and feedback can be used in order to increase speed.

In this paper, we have been able to demonstrate in many cases how chaos can be controlled successfully in 2-dimensional nonlinear discrete maps. We also hope that this technique can be applied to higher dimensional discrete maps and continuous dynamical systems.

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